Test 1: Limits
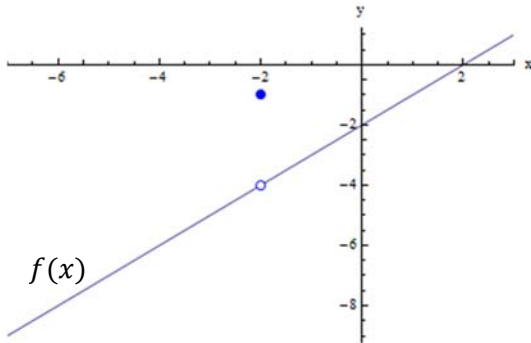
1. Draw a function $f(x)$ where $\lim\limits_{x \to 1} f(x) \neq f(1)$ (the limit as $x \to 1$ is different from the exact value of f at $x = 1$):

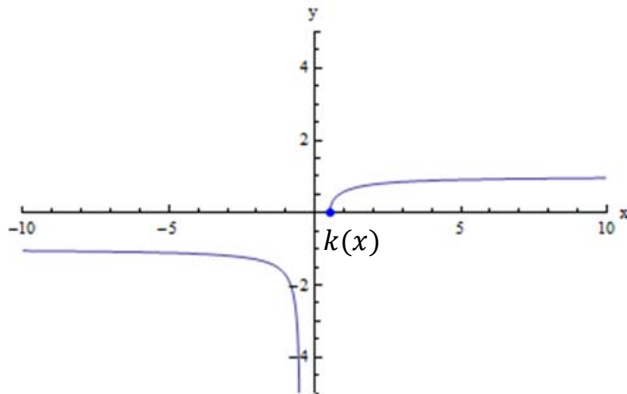2. Answer questions about each of the following functions, given the graph:
    a. $\lim\limits_{x \to -2} f(x) =$ [   ] (see graph below)
    b. On the graph below, draw another function $g(x)$ that has the **same limit** as $f(x)$ as $x \to -2$, but that is **NOT equal** to $f(x)$ for **ANY** value of x.

    c. $\lim\limits_{x \to \infty} k(x) =$ [   ] (see graph below)
    d. On the graph below, draw another function $g(x)$ that has the **same limit** as $k(x)$ as $x \to \infty$, but that is **NOT equal** to $k(x)$ for **ANY** value of x.

3. Draw two graphs, $f(x)$ and $g(x)$ so that:
    a. $\lim\limits_{x \to -2} f(x)$ does NOT exist
    b. $\lim\limits_{x \to -\infty} g(x)$ does NOT exist

4. Consider the following work that two different students did while solving a problem:

A. As $x \to -2^{-}$, $\frac{1}{x+2} \to \frac{1}{-2^{-}+2} \to \frac{1}{0^{-}} \to -\infty$

B. When $x = -2$, $\frac{1}{x+2} = \frac{1}{-2+2} = \frac{1}{0} = \infty$

a. Is all of the work from student A correct? Why or why not?

b. Is all of the work from student B correct? Why or why not?

c. The -2^{-} in A and the -2 in B represent two different ideas—explain what each of these represents and how they are different from one another.

d. The work in A uses arrows, while the work in B uses the equals sign— explain what each of these symbols represents and how they are different from one another.

5. Consider the following functions: $f(x) = 2x$, $g(x) = 2x + 1$, $h(x) = 3x$

a. Find $\lim_{x \to +\infty} [f(x) + g(x)]$

b. Find $\lim_{x \to +\infty} [f(x) + h(x)]$

c. Find $\lim_{x \to +\infty} [g(x) - f(x)]$

d. Find $\lim_{x \to +\infty} [h(x) - f(x)]$

e. Find $\lim_{x \to +\infty} [f(x) - h(x)]$

f. **Use your answers to a-e above** to help you to **explain WHY**:

    i. $\infty - \infty$ is an indeterminate form

    ii. $\infty + \infty$ is NOT an indeterminate form